VIII. *Account of recent Pendulum Operations for determining the relative Force of Gravity at the Kew and Greenwich Observatories.*

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The recent pendulum observations for the purpose of determining the gravity connexion between the Royal Observatory at Greenwich and the Royal Society's Observatory at Kew, were undertaken in order to improve and strengthen the connexion between the Indian series of pendulum operations and other series taken in other parts of the world.

The Indian series had been carried out in the years 1865 to 1873, when two invariable pendulums, the property of the Royal Society, which had been designed by Captain Kater for the purpose of investigating the relative force of gravity in different latitudes, were swung at the Kew Observatory, and at various places in and on the way to India, in the course of the operations of the Great Trigonometrical Survey of India. The work was originated at the suggestion of the President of the Royal Society, General Sir E. Sabine; the greater portion was performed by Captain J. P. Basevi, R.E., who lost his life from exposure while operating on the high tablelands of the Himalayan Mountains; the remainder was completed by Captain W. J. Heaviside, R.E.; both officers acted under the personal superintendence of General J. T. Walker, the Superintendent of the Great Trigonometrical Survey.

The points at which the pendulums were swung and the number of vibrations they made in 24 hours were determined, were mostly stations of the Central Meridional Arc of the Survey which extends from Cape Comorin to the Himalayan Mountains; a few stations were added on the East and West Coasts of India, and on neighbouring islands, and also at Aden and Ismailia. The base station of the entire series of operations—that is to say, the one at which they were commenced and concluded—was the Royal Society's Observatory at Kew, near Richmond, Surrey.

With a view to effecting a connexion between the operations in India and similar operations recently completed in Russia, and also for other reasons, two reversible pendulums, the property of the Russian Imperial Academy of Sciences, which had been employed in Russia, were sent out to India and swung at some of the Indian stations, *pari passu*, with the pendulums of the Royal Society.

For the purpose of connecting the Indian operations with those taken by KATER, SABINE, FOSTER, and other observers in various parts of the globe, it was intended to swing the Royal Society's pendulums, on their return to England, at the Royal Observatory, Greenwich, which was a station of various important series of operations. But when the time arrived, in 1873, it was found that extensive preparations were being made in the Observatory for several expeditions which were being outfitted for the observation of the approaching transit of Venus, so that no space was left available for the pendulum operations. It was therefore decided to make the desired connexion by swinging KATER's convertible pendulum, for determining the absolute length of the seconds' pendulum in any latitude, at Kew, as already it had been swung by SABINE at Greenwich. This was done by Captain HEAVISIDE, and was the last stage of the Indian pendulum operations; the results were published in vol. 5 of the 'Account of the Operations of the Great Trigonometrical Survey,' Dehra Dun, 1879, which gives full details of all the operations, including the swings with the Russian pendulums.

The absolute length of the seconds' pendulum, *in vacuo*, at Kew, was found to be 39·14008 inches of the British standard yard, in 1873, whereas at Greenwich, SABINE had found it to be 39·13734 inches of Sir GEORGE SHUCKBURGH's standard scale, in 1831.* Thus it would seem that a seconds' pendulum will make about three more vibrations in 24 hours at Kew than at Greenwich. But the two Observatories are nearly in the same latitude, and differ very little in height, and are only ten miles apart; thus this difference is much too large to be accepted as trustworthy.

In his work on Geodesy, Colonel CLARKE, C.B., R.E., of the Ordnance Survey of Great Britain, employs a large number of pendulum determinations in various parts of the globe to investigate the figure of the earth. He remarks that the selection of Kew, instead of London or Greenwich, as the base station for the Indian series of swings, was unfortunate; and, disregarding the connexion of Kew with Greenwich by the two determinations of the length of the seconds' pendulum, he employs the ratio of Madras to London from the observations of GOLDINGHAM and SABINE, and that of Kew to St. Petersburgh, by HEAVISIDE and SAWITSCH. Then he makes four different combinations of his data, from which he obtains as many values of the earth's ellipticity; and for each value he finds the corresponding system of quantities, x, indicating the apparent excess or defect of the observed over the theoretical force of gravity at each station of observation; one combination gives Kew an excess of 6·06 vibrations over London; another gives it an excess of 5·12 vibrations over Greenwich, but reduces the excess over London to 3·10 vibrations. These figures indicate variations in the vibration numbers such as are usually met with on changes of latitude of 1 to 2 degrees, and they show that the actual relation of Greenwich to

* It has recently been ascertained that very little, if any, of the difference can be due to error of the unit of the SHUCKBURGH scale as compared with the standard yard. See No. 288 of the 'Proceedings of the Royal Society' (vol. 47, 1890, p. 186.)

Kew had not yet been precisely determined, and that special observations were still required for the purpose.

In 1881, Colonel HERSCHEL, R.E., was deputed by the Secretary of State for India to take pendulum observations at the Greenwich and the Kew Observatories; also at some places in America, with a view to making a connexion with the pendulum operations of the Coast and Geodetic Survey of the United States. He employed the two pendulums of the Royal Society which had been used in the Indian operations, and also a third pendulum of precisely similar construction which had been deposited in the Kew Observatory by the Admiralty, the experience already gained in India having shown that the employment of a third pendulum was desirable. After completing his swings in England and America, he made over the three pendulums to officers of the United States' Survey, who took them round the world, and swung them at Auckland, Sydney, Singapore, Tokio, San Francisco, and finally at Colonel HERSCHEL'S station in Washington.*

When the observations came to be finally reduced, it was found that the results between Kew and Greenwich by the three pendulums were largely discordant, one giving Kew an excess of 1·97 vibrations, another an excess of 1·39, while the third gave a defect of 4·98 vibrations. It was obviously necessary that the pendulums should be again swung at the two places, in order to obtain a more satisfactory determination of the relative vibration numbers. Fresh swings were therefore made at Kew in 1888, and at Greenwich in the following year. The operations were performed by members of the Observatory staff at each place, Mr. HOLLIS taking the lead and responsibility at Greenwich, under the direction of the Astronomer Royal, and Mr. CONSTABLE at Kew, under the Superintendent of the Kew Observatory. The final results give a vibration number for Kew which differs by less than one vibration from that at Greenwich, and may be accepted as very fairly probable.

It is the object of the present paper to give an abridged account of the above operations, both the primary by Colonel HERSCHEL, and the revisionary by Messrs. HOLLIS and CONSTABLE.† For this purpose it is necessary, in the first instance, to give brief descriptions of the pendulums, and of the *modus operandi* adopted by the different observers.

Description of the Pendulums.

All three pendulums are of KATER'S Invariable Pattern; they are made of brass, with a steel knife-edge at the head of each pendulum, and they are of very nearly the same dimensions. One is numbered 4 and another 11; the third has

* Full details of the operations and their results are given in Appendix No. 14 of the 'Report of the United States Coast and Geodetic Survey' for 1884.

† Full details of Colonel HERSCHEL'S operations, in manuscript, were made over to the Royal Society for record, by the Secretary of State for India; the details of the other operations are recorded in the observatories in which they respectively took place.

no such distinguishing number, but is marked 1821, presumably the year in which it was constructed; Colonel HERSCHEL believes that it is probably No. 6 of the series, and has so called it. No. 4 was employed by SABINE in his operations between the parallels of 13° South and 80° North Latitude, in 1822–23; and No. 6 (1821) was used by the late Astronomer Royal, Sir GEORGE AIRY, in experiments in the Harton Colliery Pit, in 1854, to determine the earth's mean density; these two are the pendulums of the Royal Society which were employed throughout the operations in India. No. 11 was used by BAILEY, in London, in 1832, and by MACLEAR, at the Cape of Good Hope, in 1839; it was afterwards lent for a while to the Admiralty, and eventually deposited in the Kew Observatory.

Each pendulum is furnished with a pair of agate planes, on which it is intended to be swung. The planes are set on either side of a half-inch opening in a solid brass frame, which is mounted on a plate at the head of the receiver, and is provided with three levelling screws; outside the frame there is a pair of moveable arms carrying Y's, in which the pendulum rests while not vibrating, and on lowering which the knife edge comes in contact with the agate planes for vibration. The pendulum is placed midway between the supporting planes by hand and eye estimate, but it is always brought by the Y's down on to the same line across the planes, in all positions of the pendulum, whether the marked face is pointing towards the observer or towards the clock.

The length of the pendulum is invariable, excepting from change of temperature for which the correction to the vibration-number is known. The shape is that of a flexible bar of plate brass, 62 inches long, 1·7 inch broad, and 0·13 inch thick from the knife edge downwards for a distance of about 40 inches, where a flat circular bob, 6 inches in diameter and 1·3 inch thick, with a bevelled edge, is soldered on to the bar; the tail piece, below the bob, is reduced to a breadth of 0·7 inch, and is about 16 inches long. The bar is necessarily very flexible, its thickness being less than a tenth of its breadth, and this flexibility is greatly in contrast with the rigidity of the German and French pendulums. KATER is believed to have adopted a flexible form of bar in preference to a rigid bar designedly, under the impression that it was less likely to become permanently bent by accident, and more likely to acquire exact verticality when its knife-edge is resting on the agate planes during the course of the vibrations.

The Processes of Manipulation.

When the pendulums were sent out to India, it was intended that they should always be swung as nearly as possible in a vacuum. For this purpose a receiver of sheet copper, mounted on a substantial and well braced wooden stand, was furnished for the pendulums to be swung in; the receiver was closed above by a hemispherical glass cap, which could be removed at pleasure for the insertion or withdrawal of a pendulum. Two thermometers were fixed in a dummy pendulum, of the same size as

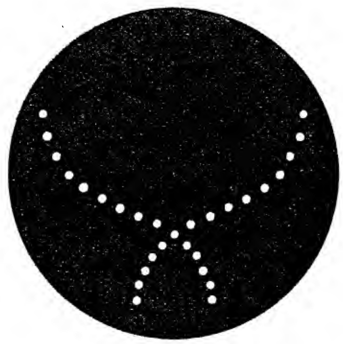
the vibrating pendulum less the tail-piece, which was fixed inside the receiver. In the Indian operations, the air was always exhausted to the lowest pressure attainable, and the vibration-number obtained at that pressure was reduced to the vacuum by special corrections, which will be afterwards described. At first the receiver was found to leak slightly, the pressure rising about an inch in the course of a day's swings; but eventually this leakage was traced to the two stuffing boxes, through which one rod passes for lowering the knife-edge on, or raising it off, the agate planes, and another rod for setting the pendulum in motion; the leaks were stopped by fitting cups round the necks of the rods, and keeping them filled with oil. When the apparatus came into Colonel HERSCHEL's hands much leakage was met with at the lower pressures. He continued to use the receiver, but he did not attempt to obtain low pressures, and he abandoned the reduction to the vacuum; his pressures ranged between 26 and 28 inches, and his observations were reduced to an adopted standard pressure of 26 inches, with the temperature of the air at 32° F. The same procedure was adopted when the pendulums were swung by the officers of the United States' Coast and Geodetic Survey. But when the apparatus was returned to England, the receiver was repaired and made thoroughly air-tight before the revisionary swings at Kew and Greenwich were commenced; then half the swings at each place were taken under a pressure of about 2 inches, and the other half at about 27 inches; the results were reduced to a vacuum in both cases, by the same formula, which will be given hereafter.

For each invariable pendulum the vibration-number is determined by swinging it in front of the pendulum of a clock and observing the times of the first and last coincidences of the two pendulums, at the beginning and end of a set of swings; also the time of a coincidence immediately following the first, in order to get an approximate value of the interval between successive coincidences; thus the number of swings made by the invariable pendulum during a given amount of clock time, is ascertained by observation, and from it the number of swings in 24 hours is calculated, after making due allowance for clock rate, arc of vibration, temperature, and pressure. The clock employed in the Indian and subsequent operations was one by SHELTON, which had been used for the same purpose by SABINE. The invariable pendulums make 200 to 300 vibrations daily less than the pendulum of a clock regulated to solar time, and about twice that number to siderial time.

For the observations of coincidence, in the Indian operations, a circular disc of white metal was mounted on the bob of the clock pendulum, and an image of it, made very slightly less in diameter than the breadth of the tail-piece of the invariable pendulum, was produced, by an intermediate lens, in the plane of the tail-piece. The image would disappear for a moment and then reappear behind the tail-piece at every apparent conjunction of the two pendulums; and these conjunctions occur when both pendulums are swinging in the same direction, the intermediate conjunctions, with the swings in opposite directions, being unobservable. The exact moment of coincidence

was at first deduced from observations of the disappearance and reappearance of both edges of the disc; but after a short time, this was considered unnecessary, and one edge only was observed, the same throughout each set of swings.

Colonel HERSCHEL substituted for the single large disc a system of multiple discs consisting of several pairs of small circles, arranged symmetrically on opposite sides of a central vertical line, and painted white on a piece of black cardboard which was attached to the bob of the clock pendulum. He designed a large variety of systems, one of which is here shown. He observed the times of disappearance and re-appearance of several pairs of discs, eventually retaining five pairs only, of which the general



mean was taken as the moment of coincidence. The United States' officers adopted one of Colonel HERSCHEL's discs, but observed on only one side of the tail-piece and not on both sides as he had done. In the revisionary operations at Kew and Greenwich, a single large disc, of which the image was made of nearly the same diameter as the tail-piece, was employed, and observations of disappearance and re-appearance were madé on one side only, as in India.*

In the Indian and the revisionary operations the times of the three first and the three last coincidences in each set of swings were observed, and the means were employed to indicate the moments of commencement and conclusion of the set; the observed intervals between successive coincidences gave the divisor to the duration of the set to find the total number of intervals which is wanted in calculating the vibration-number. In Colonel HERSCHEL's operations one or two discs were observed

* Very great precision in the determination of the moment of coincidence is unnecessary. If V be the vibration-number of a pendulum, R the clock vibrations in a mean solar day, and N the clock vibrations during a set of swings in which there are n intervals between visible coincidences, then

$$V = R\left(1 - \frac{2n}{N}\right) \quad \text{and} \quad dV = 2R\,\frac{n}{N^2}\,dN.$$

Let $R = 86{,}630$, let the duration of the set of swings be 6 hours and the interval between coincidences 6 minutes, giving $n = 60$, then

$$dV = {\cdot}022\,dN.$$

Thus an error of 4 seconds in N, which is improbably large, would not affect the vibration-number by as much as 0·1, which is but a fraction of the probable error from other causes.

immediately after the initial observations of coincidence, to obtain the interval between successive coincidences.

In the Indian operations the agate planes were always carefully levelled before the pendulum was set up on them, and the level readings, as taken before and after the swings by each pendulum, were recorded and published. As the pendulum is necessarily made to vibrate with its tail-piece a short distance in front of the scale for measuring the arc of vibration, this distance was read off on a scale fixed at right angles to the arc-scale, to enable the observed arc reading to be duly corrected; and it was measured in the two positions of the instruments, both with the marked face towards the observer and towards the clock. Thus, as the knife-edge was always lowered by the Y's down to the same line on the agate planes, and as the planes were always horizontal, half the difference between the distances of the tail-piece from the arc-scale, in the two positions, indicates the magnitude of any deviation of the bar of the pendulum, at the tail-end, from perpendicularity to the knife-edge. These distances were always recorded, and they show that both the pendulums were bent, but that the bending was practically constant throughout the whole of the operations; the marked face of Pendulum No. 4 was deflected ·5 inch outwards, and that of No. 6 (1821) ·3 inch backwards, at the tail-end. As, however, a general constancy was preserved throughout, the whole of the results were truly differential.

Colonel HERSCHEL commenced his operations at Kew by swinging No. 4 pendulum in the condition in which he found it. He soon noticed bends in both the two pendulums, and also found that the knife-edges were somewhat rusted. There was reason to suspect that the pendulums might have received some injuries when set up at one of the great Annual Exhibitions in South Kensington which was held a few years after their return from India. Consequently both the pendulums were straightened and their knife-edges were re-ground. This, of course, causes a break of continuity with the antecedent operations with these pendulums, and destroys the differentiality of the vibration numbers obtained before and afterwards.

In the revisionary operations at Kew and Greenwich the pendulums were swung in the same condition as when employed by Colonel HERSCHEL and the officers of the United States.

In the Indian operations each set of swings was usually of about 9 hours' duration, from 8 A.M. to 5 P.M., intermediate readings of the thermometers and observations of coincidence being taken at intervals of about $1\frac{1}{2}$ hour apart. When all the observations were finally reduced it was seen that, whenever the daily variation of temperature was considerable, the clock rate at different hours of the day varied sensibly from the mean daily rate; thus it was evident that the vibration-number, which depends on the actual clock rate during the set of swings, but is deduced from the mean daily rate which is derived from successive transits of the same stars, might be much influenced by variations of rate occurring during the part of the day when the pendulum is under vibration.

Colonel HERSCHEL got over this difficulty, and eliminated the influence of hourly variations of clock rate, by linking successive sets of swings together so as to fill up the whole 24 hours. He made the duration of each set of his swings somewhat less than 6 hours, taking all necessary observations of temperature, coincidence, &c. at the commencement and conclusion of the set, but without any intermediate observations; immediately after the conclusion of one set he commenced the next set. In this way observations were sometimes carried on continuously for two or three days by himself and his assistant. In the revisionary operations at Kew a similar procedure was adopted; the temperatures and coincidences in the intervals between the beginning and end of each set were generally observed also, and the temperatures when not observed were recorded on a thermograph. At Greenwich observations were made at 10 A.M., 4 P.M., and 10 P.M.; the swings at low pressure were divided into two sets of 6 hours each and one set of 12 hours, so as to fill up the 24 hours; those at high pressure into two sets of 6 hours each, to fill up 12 hours. The temperatures between the thermometer readings were registered by a thermograph. The daily range of temperature was very small at Kew, and rarely as much as 1° F. at Greenwich. At both places the time was derived electrically from the siderial standard clock at Greenwich, which is fixed in the basement of the Observatory, where there is no sensible diurnal variation of temperature. Under the actual circumstances there was no real necessity for continuous observations throughout the 24 hours to control the clock rate.

The differences in procedure and manipulation which have been pointed out thus far are not such as to have affected the results sensibly; but in one other matter there was a difference of procedure which might have materially influenced the results. Colonel HERSCHEL did not maintain the agate planes in exact horizontality; he believed that when the planes were truly level, and the pendulum was set up on them, the knife-edge, if pressed down against them by hand, was found to be not truly in contact with the planes throughout the line of bearing; consequently he dislevelled the planes until the contact, as judged by touch, was thorough, and then he commenced swinging the pendulum. Such imperfect contact was never noticed in the Indian operations; it is possible with pendulums having a rigid bar, when the knife-edge is not truly perpendicular to the bar; but with pendulums such as these, which have a very flexible bar, it seems scarcely possible, at least without a grosser displacement of the knife-edge from the perpendicular than is at all probable. The officers of the United States who swung these pendulums at several stations have been questioned on this point, and Mr. EDWIN SMITH reports that after levelling he "tested the contact of the knife-edges with the agate planes by touch and found it impracticable to make any change, so the pendulums were always swung with the planes levelled with the spirit level." In the revisionary swings this was done also.

Colonel HERSCHEL does not appear to have measured the actual dislevelment of the agate planes which was caused by his method of treatment; had he done so and his surmise been correct, the magnitude of the dislevelment would have been the same

after as it was before each transposition of the pendulum, but its sign would have changed, because its direction would have been reversed. The record of the observations gives no level readings, nor does it give the distances between the tail-piece and the arc-scale in the two positions of vibration. All that is known is that the agate planes were not maintained in a position of constant horizontality, as in all other operations. Thus the results are not strictly differential, and it is to be inferred that the large discrepancies occasionally met with between different groups of results, even when the individual results in each group are highly accordant, are due to this cause.

The results of the several operations will now be given.

Colonel Herschel's Results, reduced to temperature of 62° F., and to the Density of the Air under the pressure of 26 inches at the temperature of 32° F.

Determinations at Kew Observatory.

Preliminary Vibration-numbers of Pendulum No. 4, before straightening the bar and re-grinding the knife-edge.

Marked face, M, to front.		Plain face, P, to front.		Means.
Set.	V.	Set.	V.	
2	86158·74	15	86161·44	
3	86158·84	16	86161·02	
4	86158·05	17	86160·87	
5	86159·21	18	86161·25	
6	86157·31	19	86160·60	Face M . 86158·38
8	86158·41	20	86160·24	Face P . 86160·98
9	86158·25	21	86160·86	
10	86158·32	22	86161·26	86159·68
11	86158·31	23	86160·12	
12	86158·91	24	86160·87	
13	86158·83	25	86161·52	
14	86157·41	26	86161·69	

When this result is reduced to a vacuum, it may be compared with the results of the swings by the same pendulum at the same place which were obtained for the Indian operations. The reduction to the vacuum may be approximately taken as + 8·32, which gives the vibration-number 86168·00, to compare with 86169·45 obtained in 1864, before the pendulum was sent out to India, and 86169·57 obtained in 1873, after its return from India.

Colonel HERSCHEL'S *Determinations at Kew Observatory, continued.*

VIBRATION-NUMBERS by all three Pendulums, obtained after the bars were straightened and the knife-edges re-ground.

Pendulum No. 4.

Face M.		Face P.		Means.	
Set.	V.	Set.	V.		
74	86157·07	94	86157·16		
75	86157·62	95	86157·08		
76	86158·21	96	86157·75		
77	86157·05	97	86157·26		
78	86156·75	98	86157·54		
79	86157·74	99	86157·78		
80	86158·13	100	86158·45		
81	86157·88	101	86157·84		
82	86157·44	102	86157·27	Face M .	86157·42
83	86157·67	103	86157·52	Face P .	86157·87
84	86157·54	104	86157·65		
85	86157·06	105	86157·42		86157·64
86	86157·01	106	86157·62		
87	86156·39	107	86158·47		
88	86159·39	108	86158·87		
89	86158·01	109	86159·01		
90	86155·74	110	86158·63		
91	86157·15	111	86158·07		
92	86157·35	112	86158·21		
93	86156·94				

Colonel HERSCHEL'S *Determinations at Kew Observatory, continued.*

Pendulum No. 6 (1821).

Face M.		Face P.		Means.
Set.	V.	Set.	V.	
28	86057·21	51	86056·54	
29	86056·64	52	86055·95	
30	86056·28	53	86055·88	
31	86056·00	54	86057·15	
32	86055·05	55	86057·11	
33	86055·25	56	86056·72	
34	86055·87	57	86056·71	
35	86055·64	58	86056·78	
36	86055·82	59	86056·75	Face M . 86056·32
38	86058·82	60	86056·42	Face P . 86056·74
39	86058·11	61	86056·54	
40	86056·84	62	86057·08	86056·53
41	86056·65	63	86057·18	
42	86053·87	64	86057·18	
43	86054·77	65	86057·19	
44	86056·93	66	86057·33	
45	86056·62	67	86057·01	
46	86056·48	68	86056·93	
47	86056·52	69	86056·68	
48	86056·66	70	86057·11	
49	86056·80	71	86055·21	

Pendulum No. 11.

Face M.		Mean.
Set.	V.	
115	86099·64	
116	86099·18	
117	86099·11	
118	86099·67	
119	86100·60	
120	86099·83	
121	86101·54	
122	86100·76	
123	86099·87	
124	86100·44	Face M . 86100·57
125	86100·17	
126	86100·78	
127	86100·42	
128	86100·83	
129	86100·86	
130	86101·10	
131	86100·81	
132	86101·61	
134	86103·58	

Colonel Herschel's *Determinations at Greenwich Observatory.*

Pendulum No. 4.

Face M.		Face P.		Means.	
Set.	V.	Set.	V.		
135	86153·96	151	86156·34		
136	86153·46	152	86156·04		
137	86153·54	153	86156·24		
138	86153·10	154	86156·38	Face M, 1st Series . . .	86153·39
139	86153·99	155	86156·52	Face M, 2nd and 3rd Series	86156·65
140	86153·13	156	86156·24		
141	86153·20	157	86156·78		86155·02
142	86153·53	158	86156·49	Face P	86156·31
143	86153·35	159	86156·26		
144	86153·40	160	86155·84		86155·67
145	86153·04	161	86156·21		
146	86153·38	162	86156·58		
147	86153·29	163	86155·98		
148	86153·34	164	86156·43		
149	86153·33				
150	86153·12				
165	86156·78				
166	86156·62				
167	86157·18				
168	86156·68				
169	86157·29				
170	86157·06				
171	86157·20				
212	86156·57				
213	86156·42				
214	86156·31				
215	86156·51				
216	86156·24				
217	86156·28				
218	86156·34				
219	86156·30				

Colonel Herschel's *Determinations at Greenwich Observatory, continued.*

Pendulum No 6 (1821).

Face M.		Face P.		Means.
Set.	V.	Set.	V.	
172	86054·82	184	86054·89	
173	86055·04	185	86055·91	
174	86054·91	186	86055·70	
175	86054·81	187	86054·97	
776	86054·54	188	86055·82	Face M . 86054·87
177	86055·05	189	86055·41	Face P . 86055·41
178	86055·17	190	86055·34	
179	86054·49	191	86055·28	86055·14
180	86055·23	192	86055·74	
181	86054·79	193	86055·38	
182	86054·48	194	86055·24	
183	86055·06	195	86055·24	

Pendulum No. 11.

Face M.		Mean.
Set.	V.	
196	86104·54	
197	86105·40	
198	86105·82	
199	86104·80	
200	86105·29	
201	86105·23	
202	86105·25	
203	86105·15	
204	86105·17	Face M . 86105·55
205	86105·80	
206	86106·16	
207	86105·24	
208	86106·04	
209	86105·69	
210	86106·40	
211	86106·87	

Colonel Herschel's observations give the following values of the differences between the vibration-numbers at Kew and Greenwich.

Kew—Greenwich = + 1·97 by Pendulum No. 4.
= + 1·39 ,, No. 6 (1821).
= − 4·98 ,, No. 11

Revisionary Results, Reduced to the Temperature of 62° F., and to a Vacuum.

Mr. Constable's Determinations at Kew under Low Pressures, about 2 inches.

Pendulum No. 4.

Face M.		Face P.		Means.
Set.	V.	Set.	V.	
4 5 6	86167·56 86167·07 86167·31	1 2 3 7 8 9	86167·12 86167·47 86167·36 86165·91 86166·53 86165·73	Face M . 86167·31 Face P . 86166·69 86167·00

Pendulum No. 6 (1821).

Face M.		Face P.		Means.
Set.	V.	Set.	V.	
10 11 12	86066·74 86065·97 86066·04	13 14 15 16	86067·15 86067·00 86067·58 86066·83	Face M . 86066·25 Face P . 86067·14 86066·70

Pendulum No. 11.

Face M.		Face P.		Means.
Set.	V.	Set.	V.	
17 18 19	86117·41 86117·11 86117·78	20 21 22	86116·82 86117·24 86117·44	Face M . 86117·43 Face P . 86117·17 86117·30

Mr. CONSTABLE's Determinations at Kew under High Pressures, about 27 inches.

Pendulum No. 4.

Face M.		Face P.		Means.	
Set.	V.	Set.	V.		
39	86165·20	43	86165·75	Face M .	86164·80
40	86164·81	44	86165·70	Face P .	86165·74
41	86164·44	45	86165·73		
42	86164·75	46	86165·80		86165·27

Pendulum No. 6 (1821).

Face M.		Face P.		Means.	
Set.	V.	Set.	V.		
31	86065·98	35	86065·80	Face M .	86066·07
32	86066·22	36	86065·57	Face P .	86065·54
33	86066·24	37	86065·43		
34	86065·84	38	86065·35		86065·80

Pendulum No. 11.

Face M.		Face P.		Means.	
Set.	V.	Set.	V.		
23	86115·88	27	86115·82	Face M .	86116·14
24	86116·25	28	86115·92	Face P .	86115·94
25	86116·10	29	86116·04		
26	86116·33	30	86115·97		86116·04

Mr. Hollis's Determinations at Greenwich under Low Pressures, about 2 inches.

Pendulum No. 4.

Face M.		Face P.		Means.	
Set.	V.	Set.	V.		
1	86165·08	5	86165·24	Face M .	86165·06
2	86165·03	6	86165·25	Face P .	86165·16
3	86165·09	7	86165·10		86165·11
4	86165·02	8	86164·98		
		9	86165·25		

Pendulum No. 6 (1821).

Face M.		Face P.		Means.	
Set.	V.	Set.	V.		
10	86065·61	16	86065·60	Face M .	86065·64
11	86065·47	17	86065·17	Face P .	86065·47
12	86065·55	18	86065·58		86065·56
13	86065·56	19	86065·54		
14	86065·71	20	86065·51		
15	86065·91	21	86065·43		

Pendulum No. 11.

Face M.		Face P.		Means.	
Set.	V.	Set.	V.		
22	86116·73	27	86116·22	Face M .	86116·68
23	86116·65	28	86116·41	Face P .	86116·30
24	86116·72	29	86116·39		86116·49
25	86116·79	30	86116·18		
26	86116·50				

MR. HOLLIS's determinations at Greenwich under High Pressures, about 27 inches.

Pendulum No. 4.

Face M.		Face P.		Means.
Set.	V.	Set.	V.	
39 40 41 42	86163·84 86163·98 86164·30 86164·33	43 44 45 46 47	86163·99 86163·78 86164·07 86164·00 86164·03	Face M . 86164·11 Face P . 86163·97 86164·04

Pendulum No. 6 (1821).

Face M.		Face P.		Means.
Set.	V.	Set.	V.	
48 49 50 51	86064·47 86064·45 86063·93 86064·09	52 53 54 55	86063·77 86063·74 86063·55 86063·54	Face M . 86064·23 Face P . 86063·65 86063·94

Pendulum No. 11.

Face M.		Face P.		Means.
Set.	V.	Set.	V.	
35 36 37 38	86116·07 86116·26 86116·17 86116·07	31 32 33 34	86115·20 86115·21 86115·21 86115·26	Face M . 86116·14 Face P . 86115·22 86115·68

Thus the revisionary operations give the following values of the differences between the vibration-numbers at Kew and Greenwich :—

	Low pressures.	High pressures.		
Kew—Greenwich	= + 1·89	+ 1·23	by pendulum	No. 4.
	= + 1·14	+ 1·86	„	No. 6 (1821).
	= + 0·81	+ 0·36	„	No. 11.
General mean	= + 1·22 ± ·16.			

It will be seen that the mean value is fairly in accordance with the values derived from Colonel HERSCHEL'S observations with pendulums No. 4 and No. 6 (1821). His swings with those pendulums, at Kew, were made in the basement of the Kew Observatory, within a few feet of the spot at which the revisionary swings, with all three pendulums, were made; but he swung pendulum No. 11 in a shed outside the Observatory, under circumstances of great disadvantage as regards the stability and firmness of the support of the stand of the invariable pendulum and also of the support of the clock. Thus his observations at Kew, with pendulum No. 11, though generally very accordant *inter se*, are very probably burdened with a large constant error, and must therefore be rejected.

On the Reduction to a Vacuum.

In all pendulum experiments—even those of a purely differential character, as with invariable pendulums—it has been generally customary to apply a correction for the retardation which is caused by the air, in order to obtain results such as would have been obtained if the pendulum had been swung in a vacuum. This correction was originally determined by calculating the influence of the buoyancy of the atmosphere in diminishing the weight—and consequently the accelerating force—of the pendulum. Afterwards BESSEL showed that the correction thus obtained was too small, for the pendulum is accompanied in its oscillation by a certain amount of air, varying with its form, which increases the mass in vibration and the moment of inertia. Thus the buoyancy correction has to be multiplied by a factor, $1 + k$, which can be computed mathematically for pendulums of certain simple forms, but must be determined experimentally, by swings at high and low pressures, when the form is not susceptible of being brought under mathematical treatment. The buoyancy correction, thus augmented, is usually called the pressure correction.

The buoyancy correction and the pressure correction have been investigated for pendulums No. 4 and No. 6 (1821) by special and laborious series of operations which are fully set forth in vol. 5 of the 'Account of the Operations of the Great Trigonometrical Survey of India.' Nothing of the kind is known to have been done for No. 11; but the results obtained for the two first pendulums are so closely accordant that they may be applied without objection to the third, which is almost identical with them in form and construction.

The buoyancy correction $= \cdot 23 \dfrac{\beta}{1 + \cdot 0023\,(t - 32°)}$ in which β is the pressure in inches, and t the temperature in degrees of FAHRENHEIT.

The pressure correction was found to be $\cdot 32 \dfrac{\beta}{1 + \cdot 0028\,(t - 32°)}$ by experimental swings which were made specifically for the purpose at Kew, under extreme high and low pressures, immediately before the pendulums were sent out to India. Corrections determined by this formula were applied, provisionally, to the whole of the

swings in India; and this has also been done to the revisionary swings at Kew and Greenwich, to produce the vibration-numbers which have already been set forth.

But, in the course of the operations in India, Captain BASEVI reinvestigated both the temperature and the pressure corrections of his two pendulums, those of No. 4 with great elaboration. A series of several sets of swings was made with it at each of the successive pressures of 0·6, 1·9, 4·2, 10·0, 17·5, and 27·7 inches, at the temperature of about 101° F; another series at the same pressures, at the temperature of about 53°; and a third at the pressures of 1·9 and 4·2 inches and temperature of about 80°. He came to the conclusion that the pressure correction is best represented by an empirical formula of three terms,

$$A\beta^{\frac{1}{2}} + B\beta + C\beta^{\frac{3}{2}},$$

in which the second term is the correction for buoyancy. Then he assumed A to be $= x\sqrt{461^\circ + t}$ and $C = y \div \sqrt{461^\circ + t}$ — 461° being the absolute zero of the air thermometer—and formed a corresponding series of equations for the determination of x and y from his fourteen sets of observations. The solution of these equations gave $x = \cdot 022 \pm \cdot 002$, and $y = \cdot 123 \pm \cdot 025$, which values satisfied the equations of condition very satisfactorily.

But the subject is one of great complexity and difficulty, as will be seen in consulting Chapter VI. of the Indian pendulum volume. Something appears to be wanting to explain the inconsistencies between vibration-numbers derived from different series of very accordant observations which are occasionally met with. Possibly it may be necessary to take cognisance of the atmospheric humidity during the observations, which has never been done hitherto. Or it may be that the inconsistencies arise from changes in the relative conditions of the bearing surfaces of the knife-edge and the agate planes, which are met with on successive transpositions of the pendulum, and which the observer cannot control.

Transposition is almost invariably attended with a change in the vibration-number; but in the Indian operations it was found that the changes were not constant in either sign or magnitude; it is shown, at page 114 of the volume already cited, that, for the whole of the 34 stations of observation, the mean value of M — P ranges from + ·54 to — ·52, and has an average value of + ·07 ± ·03 for Pendulum No. 4, and ranges from + ·67 to — ·59, with the average value — ·04 ± ·03 for Pendulum No. 6 (1821).

In reducing the Indian swings for investigating the pressure correction, Captain BASEVI'S observations of the vibration-numbers at different pressures were employed directly, without having recourse to his empirical formula. The observed vibration-numbers at each pressure were reduced to a vacuum by the Kew formula with the numerical constant ·32, and then the results for the higher pressures were compared with the result for the lowest pressure, which, being 0·6 of an inch, was very close to the vacuum; and it was found that the higher pressures required residual positive corrections, increasing with the temperature as well as the pressure; at the highest

pressure, 27·7 inches, the correction amounted to ·53 under the temperature of 53° and to 1·34 under the temperature of 100°. Corresponding corrections were therefore applied to the whole of the Indian swings, as they had already been provisionally reduced by the Kew formula.

This must now be done for the revisionary swings at Kew and Greenwich, the results of which, as hitherto presented, have been reduced to the vacuum by the Kew formula only. It will be seen that the vibration-numbers at the pressure of 27 inches are less than those at the pressure of 2 inches by 1·30 at Kew and 1·16 at Greenwich; but it appears from Captain BASEVI'S investigations that the high pressure results at both places should be increased by about 0·7, which will reduce the discordances with the low pressure results to 0·60 and 0·46.

When pendulum operations are differential, and the variations of atmospheric pressure, at different stations, are small, the value of the correction for pressure does not require to be known with much accuracy. Reducing to a vacuum is also then unnecessary, and any convenient standard of atmospheric density may be employed instead. Thus Colonel HERSCHEL has sufficiently provided for the elimination of the effects of variations of pressure at his stations, which were all of low elevation, by reducing his swings to the standard pressure of 26 inches under the temperature of 32° F., instead of to a vacuum.

The Indian swings were invariably reduced to a vacuum, in accordance with previous procedure. They were made in an exhausted receiver, under the lowest pressure attainable, partly because this enabled them to be extended over a longer time, and thus be less influenced by hourly variations of clock-rate, than if taken under full pressure; partly because the receiver would protect the pendulum from the action of currents of air; and partly to obtain as nearly a uniform pressure at all the stations as possible, and thus secure strictly differential results; for it was intended that the pendulums should be swung both at low levels in the neighbourhood of the ocean and at the highest attainable elevations, as on the table-lands of the Himalayan mountains, where the pressure of the atmosphere is halfway down to the vacuum, so that a considerable range of pressure had to be met with and provided for in the best way possible. By exhausting the receiver, the pressures under which the swings were actually taken were generally maintained between 1 and 2 inches, excepting at first, when slight leakage was met with, the locus of which was not immediately detected, and then the pressures ranged from 1 to 4 inches. But these differences of pressure are so small that the uncertainty as to the precise amount of the pressure correction cannot exert an appreciable influence on the differential results which have already been deduced, and which are the ultimate object of the observations. And this is the case also both in Colonel HERSCHEL'S and in the revisionary operations, the range of pressure being always under 2 inches, whether the swings were taken under high pressures only or under both high and low pressures.

Thus, a more exact knowledge of the correction for pressure might sensibly affect

the vibration-numbers for Kew and Greenwich which have already been presented; but it would not affect the differences between those numbers, which are what is really wanted, to a degree that is at all comparable with the errors to which pendulum observations are liable from other causes.

Reduction to the Sea-level.

The pendulums were swung at an elevation above the mean sea-level of about 15 feet at Kew and 157 feet at Greenwich. The vibration-numbers must be correspondingly increased.

The well-known formula for the correction for height is

$$\text{Correction} = \mathrm{V}\frac{h}{r},$$

V being the vibration-number, h the height, and r the radius of the earth. Dr. Young has suggested that account should also be taken of the continental mass which is situated between the level of the sea and that of the station of observation, in increasing the force of attraction and consequently the vibration-number. Thus, in accordance with his views, the usually-accepted correction takes cognisance of both height and mass, and is

$$= \frac{5}{8}\mathrm{V}\frac{h}{r}.$$

Thus for these pendulums, when h is expressed in feet, we have

$$\text{Correction for height only} = \frac{h}{243},$$

$$\text{Correction for height and mass} = \frac{h}{391}.$$

It is now, however, very questionable whether any reduction for mass is allowable. The pendulum operations in India have thrown much light on the constitution of the earth's crust, and shown that there is a marked deficiency of density under the Himalayan mountains, and an increase of density under the bed of the Indian Ocean. Thus continental matter above the sea level may be conceived as appertaining to the strata underneath, immediately below the sea level, which are correspondingly attenuated. In this case, the excess of matter above would probably compensate for the deficiency of matter below, and would not form an attracting force to be independently allowed for while no cognisance is taken of the deficiency below.

If the vibration-numbers at Kew and Greenwich are corrected for height only, the correction to be applied to their difference will be

$$= \frac{15 - 157}{243} = -0{\cdot}58,$$

whence we obtain

$$\text{Kew—Greenwich} = 1{\cdot}22 - 0{\cdot}58 = 0{\cdot}64$$

as the result of the revisionary operations.

ADDENDUM.

RESULTS of the swings at other stations which were taken with the same pendulums, by Colonel HERSCHEL, and by Mr. EDWIN SMITH of the United States Coast and Geodetic Survey. They are reduced to the temperature of 62° F., and to the density of air under the pressure of 26 inches at the temperature of 32° F.; but they are not reduced to the sea-level.

Swings by Colonel HERSCHEL.

Stations.	Height in feet above sea.	No. 4.	No. 6 (1821).	No. 11.
London,* Langham Place, Cellar .	85	86157·31	86055·97	86109·59
Washington, Smithsonian Institute	34	86109·45	86008·35	86060·93
Hoboken, Stevens Institute . . .	30	86115·23	86014·06	86066·93
Whence				
Greenwich—London . . . . . =	..	+ 1·64	+ 0·83	+ 4·05
London—Washington . . . . =	..	+ 47·86	+ 47·62	+ 48·66
Washington—Hoboken . . . =	..	− 5·78	− 5·71	− 6·00

Swings by Mr. EDWIN SMITH.

Stations.	Height in feet.	No. 4.	No. 6 (1821).	No. 11.
Auckland	261	86102·75	86002·11	86054·13
Sydney	140	86090·93	85990·32	86042·08
Singapore	45	86021·13	85919·97	85971·34
Tokio†	20	86099·83	85995·17	86046·91
San Francisco	375	86103·77	86003·13	86055·66
Washington	34	86109·31	86009·29	86061·41
Whence				
Auckland—Sydney . . . . . =	..	+ 11·82	+ 11·79	+ 12·05
Sydney—Singapore . . . . . =	..	+ 69·80	+ 70·35	+ 70·74
Singapore—Tokio . . . . . . =	..	− 78·70	− 75·20	− 75·57
Tokio—San Francisco . . . . =	..	− 3·94	− 7·96	− 8·75
San Francisco—Washington . =	..	− 5·54	− 6·16	− 5·75

* This station is at No. 1, All Souls Place, Langham Place; it is about 380 feet S.E. of, and 14 feet lower in level than, Mr. BROWNE'S house in Portland Place, which was KATER'S station. The value of Greenwich—London (Portland Place) = + 0·48, with Invariable Pendulum No. 12; it was determined by SABINE in 1828, see 'Phil. Trans.' for 1829, p. 87.

† Mr. EDWIN SMITH remarks of the observations at Tokio, that "the vibration-number of Pendulum No. 4 is between three and four vibrations too great. I can only explain this discrepancy by the supposition that some foreign material was adhering to the pendulum during these observations. Great care was always taken in wiping the pendulums before suspending them."